

# Experimental bounds on sterile neutrino mixing angles

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## Abstract

We derive bounds on the mixing between the left-chiral (“active”) and the right-chiral (“sterile”) neutrinos, provided from the combination of neutrino oscillation data and direct experimental searches for sterile neutrinos. We demonstrate that the mixing of sterile neutrinos with *any* flavour can be significantly suppressed, provided that the angle  $\theta_{13}$  is non-zero. This means that the lower bounds on sterile neutrino lifetime, coming from the negative results of direct experimental searches can be relaxed (by as much as the order of magnitude at some masses).

We also demonstrate that the results of the negative searches of sterile neutrinos with PS191 and CHARM experiments are not applicable directly to the see-saw models. The reinterpretation of these results provides up to the order of magnitude stronger bounds on sterile neutrino lifetime than previously discussed in the literature. We discuss the implications of our results for the Neutrino Minimal Standard Model (the  $\nu$ MSM).

## 1 Introduction

Transitions between neutrinos of different flavours (see e.g. [1] for a review and Refs. [2, 3] for the recent update of experimental values) are among the few firmly established phenomena *beyond the Standard Model of elementary particles*. The simplest explanation is provided by the “neutrino flavour oscillations” – non-diagonal matrix of neutrino propagation eigenstates in the weak charge basis. While the absolute scale of neutrino masses is not established, particle physics measurements put the sum of their masses below 2 eV [4] while from the cosmological data one can infer an upper bound of 0.58 eV at 95% CL [5].

A traditional explanation of the smallness of neutrino masses is provided by the *see-saw mechanism* [6–9]. It assumes the existence of several *right-handed neutrinos*, coupled to their Standard Model (SM) counterparts via the Yukawa interaction, providing the Dirac masses,  $M_D$ , for neutrinos. The Yukawa interaction terms dictate the SM charges of the right-handed particles: they turn out to carry no electric, weak and strong charges; therefore they are often termed “singlet,” or “sterile” fermions. Sterile neutrinos can thus have Majorana masses,  $M_s$ , consistent with the gauge symmetries of the Standard Model. If the Majorana masses are

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much larger than the Dirac ones, the *type I seesaw formula* [6–9] holds, expressing the mass matrix of observed neutrinos ( $\mathcal{M}_\nu$ ) via

$$\mathcal{M}_\nu = -M_D M_s^{-1} M_D^T, \quad (1)$$

where  $\mathcal{M}_\nu$  is a  $3 \times 3$  matrix of active neutrino masses, mixings, and (possible) CP-violating phases. The masses of sterile neutrinos are given by the eigenvalues of their Majorana mass matrix (with the corrections of the order  $M_D^2/M_s^2$ ). They are much heavier than the active neutrino masses as a consequence of (1).

Numerous searches for sterile neutrinos in the mass range up to  $\sim 100$  GeV had been performed (see the corresponding section in Particle Data Group [4],<sup>1</sup> see also [10] and refs. therein). These searches provided upper bounds on the strength of interaction of these neutral leptons with the SM neutrinos of different flavours – active-sterile neutrino *mixing angles*  $\vartheta_\alpha^2 \propto \left| \frac{M_{D,\alpha}}{M_s} \right|^2$  for sterile neutrino with the mass  $M_s$ .<sup>2</sup> These bounds then can be interpreted as *lower bounds* on the lifetime of sterile neutrinos  $\tau_s$  via

$$\tau_s^{-1} = \frac{G_F^2 M_s^5}{96\pi^3} \sum_X \vartheta_\alpha^2 B_X^{(\alpha)}, \quad (2)$$

where the sum runs over various particles to which sterile neutrino can decay, depending on their mass ( $\nu$ ,  $e^\pm, \mu^\pm, \tau^\pm$ ,  $\pi$ ,  $K$ , heavier mesons and baryons) and dimensionless quantities  $B_X^\alpha$  depend on the branching ratios (see Appendix A for details). The lower bound on the lifetime  $\tau_s$  is usually dominated by the least constrained mixing angle,  $\vartheta_\tau^2$  (as will be shown later).

This bound can be made *stronger* if one assumes that the same particles are also responsible for the neutrino oscillations. The see-saw formula (1) allows to fix (at least partially) the ratio of the mixing angles  $\vartheta_\alpha^2/\vartheta_\beta^2$  of different flavours. In the simplest case when only two sterile neutrinos are present (the minimal number, required to explain two observed neutrino mass differences) the ratio of all three mixing angles can be fixed, which makes the bounds on the lifetime significantly stronger (essentially, they are determined now by the *strongest*, rather than the weakest direct bound on  $\vartheta_\alpha$ ). When confronted with the cosmological bounds (e.g. from Big Bang Nucleosynthesis [11, 12]) they seem to close the window of parameters for sterile neutrinos with the mass lighter than about 150 MeV [13, 14].

In this paper we demonstrate that the bounds on sterile neutrino lifetime from accelerator searches can be relaxed even in the case when there are only two sterile neutrinos, responsible for the observed neutrino oscillations. Namely, we show that the recent neutrino oscillation data [3] allow for such a choice of the active-sterile Yukawa couplings that the mixing of sterile neutrinos with any given flavour can be strongly suppressed (provided that the active neutrino mixing angle  $\theta_{13} \neq 0$ , as hinted recently by MINOS [15] and T2K [16] experiments). This changes the restrictions on the sterile neutrino lifetime in some cases by as much as 10 times in the normal hierarchy of active neutrino masses and two times in the inverted hierarchy. The results of this paper partially overlap with [17] (also [18]), and we compare in the corresponding places to the previous works.

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<sup>1</sup><http://pdglive.lbl.gov/Rsummary.brl?nodein=S077&inscript=Y>

<sup>2</sup>Here and below we use the letter  $\vartheta$  for *active-sterile mixing angles* (defined by Eq. (11) below) while reserving  $\theta_{12}, \theta_{13}$  and  $\theta_{23}$  for the measured parameters of the active neutrinos matrix  $\mathcal{M}_\nu$ . These quantities  $\vartheta_\alpha$  are often denoted  $|V_{4\alpha}|^2$  or  $|U_{x\alpha}|^2$  in the experimental papers, to which we refer. Here and below the Greek letters  $\alpha, \beta$  are flavour index  $e, \mu, \tau$  and  $i, j = 1, 2, 3$  denote active neutrino mass eigenstates.

The paper is organized as follows: in Section 2 we briefly describe the type I see-saw model we use. We then analyze the see-saw equations and demonstrate that the mixing with any flavour  $\vartheta_\alpha^2$  can become suppressed and investigate relations between different mixing angles imposed by the see-saw mechanism (Section 3). Section 4 is devoted to the overview of the experiments, searching for sterile neutrinos with the masses below 2 GeV, and the way one should interpret their results to apply to the see-saw models that we study. Section 5 summarizes our revised bounds on mixing angles and translates them into the resulting constraints on sterile neutrino lifetime (Figs. 6). We conclude in Section 6, discussing possible implications of our results.

## 2 Sterile neutrino Lagrangian

The minimal way to add sterile neutrinos to the Standard Model is provided by the Type I see-saw model [6–9] (see also [19] and refs. therein):

$$\Delta\mathcal{L}_N = \sum_{I,J=1}^{\mathcal{N}} i\bar{N}_I \partial_\mu \gamma^\mu N_I - \left( F_{\alpha I} \bar{L}_\alpha N_I \tilde{\Phi} - \frac{(M_s)_{IJ}}{2} \bar{N}_I^c N_J + h.c. \right), \quad (3)$$

where  $F_{\alpha I}$  are new Yukawa couplings,  $\Phi_i$  is the SM Higgs doublet,  $\tilde{\Phi}_i = \epsilon_{ij} \Phi_j^\dagger$ . This model is renormalizable, has the same gauge symmetries as the Standard Model, and contains  $\mathcal{N}$  additional Weyl fermions  $N_I$  — sterile neutrinos ( $N_I^c$  being the charge-conjugate fermion, in the chiral representation of Dirac  $\gamma$ -matrices  $N_I^c = i\gamma^2 N_I^\dagger$ ). The number of these singlet fermions must be  $\mathcal{N} \geq 2$  to explain the data on neutrino oscillations. In the case of  $\mathcal{N} = 2$  there are *11 new parameters* in the Lagrangian (3), while the neutrino mass/mixing matrix  $\mathcal{M}_\nu$  has 7 parameters in this case. The situation is even more relaxed for  $\mathcal{N} > 2$ . The see-saw formula (1) does not allow to fix the scale of Majorana and Dirac  $M_{D,\alpha I} = F_{\alpha I} \langle \Phi \rangle$  masses. In this work we will mostly concentrate on sterile neutrinos with their masses  $M_s$  in the MeV–GeV range – the range that is probed by the direct searches. To further simplify our analysis we will concentrate on the case when the masses of both sterile neutrinos are close to each other (so that  $\Delta M \ll M_s$ ). One important example of such model is provided by the *Neutrino Extended Standard Model* (the  $\nu$ MSM) ([20, 21], see [14] for review). The  $\nu$ MSM model contains 3 sterile neutrinos, whose masses are roughly of the order of those of other leptons in the Standard Model. Two of these particles (approximately degenerate in their mass) are responsible for baryogenesis and neutrino oscillations and the third one is playing the role of dark matter. The requirement of dark matter stability on cosmological timescales makes its coupling with the Standard Model species so feeble, that it does not contribute significantly to the neutrino oscillation pattern [22]. Therefore, when analyzing neutrino oscillations, the  $N_1$  can be omitted from the Lagrangian and index  $I$  in the sums runs through 2 and 3 only:

$$\mathcal{L}_{see-saw} = \mathcal{L}_{SM} + \bar{N}_I i \partial_\mu \gamma^\mu N_I - M_{D,\alpha I} \bar{\nu}_\alpha N_I - M_{D,\alpha I}^* \bar{N}_I \nu_\alpha - M_s \bar{N}_2^c N_3 - M_s \bar{N}_3 N_2^c. \quad (4)$$

This parametrization coincides with [23, Eq. (2.1)]. Note that we use basis of singlet neutrinos where the mass matrix is off-diagonal. Recent computation of the baryon asymmetry of the Universe in the  $\nu$ MSM [24] demonstrated that sterile neutrinos with the mass as low as several MeV can be responsible for baryogenesis and neutrino oscillations within the  $\nu$ MSM.

This prompts us to re-analyze the implication of negative direct experimental searches for the Yukawa couplings of sterile neutrinos with the MeV masses. We limit our analysis by  $M_s \leq 2$  GeV, as for the higher masses the existing experimental bounds do not probe the region of mixing angles, required to produce successful baryogenesis in the  $\nu$ MSM [24].

### 3 Solution of the see-saw equations

In this Section we investigate how mixing angles between active and sterile neutrinos are related to parameters of the observable neutrino matrix  $\mathcal{M}_\nu$ . We will demonstrate that the mixing angle  $\vartheta_e^2$  in the case of normal hierarchy and the mixing angles  $\vartheta_\mu^2$  or  $\vartheta_\tau^2$  in the case of inverted hierarchy, can become suppressed as we vary the parameters of the neutrino matrix away from their best-fit values (but within the experimentally allowed  $3\sigma$  bounds).

#### 3.1 Parametrization of the Dirac mass matrix

We use the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) parametrization of the neutrino matrix  $\mathcal{M}_\nu$  (see e.g. Eqs.(2.10) and (2.12) of [1])

$$\mathcal{M}_\nu = V^* \text{diag}(m_1 e^{-2i\zeta}, m_2 e^{-2i\xi}, m_3) V^\dagger, \quad (5)$$

where  $V$  is the unitary matrix, whose explicit form is reminded in Appendix B. Redefining a Dirac mass matrix as<sup>3</sup>

$$M_D \rightarrow \tilde{M}_D \equiv V^T M_D, \quad (6)$$

we can rewrite the see-saw relation (1) in the following form:

$$\text{diag}(m_1 e^{-2i\zeta}, m_2 e^{-2i\xi}, m_3)_{ij} = -\frac{\tilde{M}_{D,i2}\tilde{M}_{D,j3} + \tilde{M}_{D,i3}\tilde{M}_{D,j2}}{M_s}, \quad (7)$$

The rank of the active neutrino mass matrix  $\mathcal{M}_\nu$  is 2 in the case of two sterile neutrinos, meaning that one of the masses  $m_i$  is zero. Two choices of “hierarchies” of the mass eigenvalues are possible. The first one is called *normal hierarchy* (NH) and corresponds to  $0 \leq m_1 < m_2 < m_3$ . The second one is called *inverted hierarchy* (IH) and is realized for  $0 \leq m_3 < m_1 < m_2$ .

Once the mass  $M_s$  is fixed, the solutions of Eq. (7) contain one unknown complex parameter,  $z$ . Its presence reflects a symmetry of the see-saw relation (7) under the change  $(\tilde{M}_{D,i2}, \tilde{M}_{D,i3}) \rightarrow (z\tilde{M}_{D,i2}, z^{-1}\tilde{M}_{D,i3})$  [25]. It is this freedom that does not allow to fix the absolute scale of  $\tilde{M}_D$  (i.e. the value of  $\vartheta^2$ ) even if  $M_s$  is chosen.

The change  $z \rightarrow z^{-1}$  is equivalent to the redefinition of  $N_2 \rightarrow N_3$ ,  $N_3 \rightarrow N_2$  together with shift of the Majorana phase  $\xi \rightarrow \xi + \pi$  in (7). Therefore in subsequent analysis we will choose  $|z| \geq 1$  without the loss of generality.

#### 3.2 Normal hierarchy

For normal hierarchy the explicit see-saw relation is

$$\text{diag}(0, m_2 e^{-2i\xi}, m_3)_{ij} = -\frac{\tilde{M}_{D,i2}\tilde{M}_{D,j3} + \tilde{M}_{D,i3}\tilde{M}_{D,j2}}{M_s}. \quad (8)$$

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<sup>3</sup>The Dirac matrix  $\tilde{M}_D$  has indexes  $I = 2, 3$  and  $i, j = 1, 2, 3$  – neutrino propagation basis

Normal hierarchy		Inverted hierarchy	
$\Delta m_{21}^2$	$(7.09 - 8.19) \times 10^{-5} \text{ eV}^2$	$\Delta m_{13}^2$	$(2.13 - 2.67) \times 10^{-3} \text{ eV}^2$
$\Delta m_{31}^2$	$(2.14 - 2.76) \times 10^{-3} \text{ eV}^2$		
$\sin^2 \theta_{12}$	$0.27 - 0.36$		
$\sin^2 \theta_{23}$	$0.39 - 0.64$		
$\sin^2 \theta_{13}$	$0.001 - 0.035$	$\sin^2 \theta_{13}$	$0.001 - 0.039$

Table 1: The  $3\sigma$  bounds on the parameters of the mass matrix  $\mathcal{M}_\nu$ , adopted from [3]. Here  $\Delta m_{ij}^2 = m_i^2 - m_j^2$ . The boundaries for inverted hierarchy as the same as for the normal one, unless written explicitly.

Diagonal components of this matrix equation give

$$\tilde{M}_{D,12}\tilde{M}_{D,13} = 0, \quad \tilde{M}_{D,22}\tilde{M}_{D,23} = \frac{1}{2}m_2M_s e^{-2i\xi}, \quad \tilde{M}_{D,32}\tilde{M}_{D,33} = \frac{1}{2}m_3M_s. \quad (9)$$

Using  $m_2, m_3 \neq 0$  we find that  $\tilde{M}_{D,22}, \tilde{M}_{D,23}, \tilde{M}_{D,32}, \tilde{M}_{D,33}$  are *all* non-zero. Analysis of non-diagonal terms reveals that *both*  $\tilde{M}_{D,12}$  and  $\tilde{M}_{D,13}$  are zero and there are two general solutions (c.f. [25]):

$$\tilde{M}_{D,i2}^\pm = iz\sqrt{\frac{M_s}{2}}(0, \pm ie^{-i\xi}\sqrt{m_2}, \sqrt{m_3}), \quad \tilde{M}_{D,i3}^\pm = iz^{-1}\sqrt{\frac{M_s}{2}}(0, \mp ie^{-i\xi}\sqrt{m_2}, \sqrt{m_3}). \quad (10)$$

The solution  $\tilde{M}_D^+$  with  $\xi = \psi + \pi$  equals to  $\tilde{M}_D^-$  with  $\xi = \psi$ . It allows us to consider only one solution  $\tilde{M}_D^+$  on the interval  $0 \leq \xi < 2\pi$ . In what follows we therefore omit the superscript +.<sup>4</sup>

The mixing angles of the active-sterile neutrinos are defined as follows:

$$2\vartheta_\alpha^2 \equiv \sum_I |(M_D M_s^{-1})_{\alpha I}|^2 = \sum_I |(V^* \tilde{M}_D M_s^{-1})_{\alpha I}|^2 = \frac{1}{M_s^2} \sum_I |(V^* \tilde{M}_D)_{\alpha I}|^2. \quad (11)$$

Inserting the explicit solution (10) for  $\tilde{M}_D$  results in

$$\vartheta_\alpha^2 = \frac{|z|^2}{4M_s} |\sqrt{m_3}V_{\alpha 3} - ie^{i\xi}\sqrt{m_2}V_{\alpha 2}|^2 + \frac{1}{4M_s|z|^2} |\sqrt{m_3}V_{\alpha 3} + ie^{i\xi}\sqrt{m_2}V_{\alpha 2}|^2. \quad (12)$$

For  $|z| \gg 1$  the contribution of  $\tilde{M}_{D,i3}$  is suppressed compared with that of  $\tilde{M}_{D,i2}$  and therefore we neglect the former (we will comment below on the case  $|z| \gtrsim 1$ ).

As the value of the Majorana phase  $\xi$  is undetermined experimentally, the condition  $\vartheta_\alpha = 0$  is satisfied iff  $m_3|V_{\alpha 3}|^2 = m_2|V_{\alpha 2}|^2$  (we neglect second term on the r.h.s. of (12)). For the electron flavour ( $\alpha = e$ ) it translates into

$$\sin^2 \theta_{12} \frac{m_2}{m_3} = \tan^2 \theta_{13}, \quad (13)$$

<sup>4</sup>Unlike the parametrizations used e.g. in Ref. [17, 25, 26] this way of parametrizing the solution of the see-saw equations shows that there is only one branch of solutions, with all other related to it via redefinitions  $N_2 \leftrightarrow N_3$  and shift of the Majorana phases. In particular in the parametrization we used it is much easier to analyze whether mixing angles become zero. The relation  $|z| = \exp(\text{Im } \omega)$  holds, where the parameter  $\omega$  was employed in [17].

which, in principle, can be satisfied only for non-zero  $\theta_{13}$ . This result has been already obtained in [17].

The bounds on the parameters of the mass matrix  $\mathcal{M}_\nu$  at the  $3\sigma$  level that we use are shown in Table 1. Note that in this paper we do not take into account statistical correlations between different oscillation parameters, allowing them to vary independently within their  $3\sigma$  intervals. Consequently, we obtain the  $3\sigma$  intervals for the combinations of parameters, entering Eq. (13):

$$\begin{aligned} 0.043 &< \sin^2 \theta_{12} \frac{m_2}{m_3} < 0.070, \\ 0.001 &< \tan^2 \theta_{13} < 0.035. \end{aligned} \quad (14)$$

It means that the relation (13) *does not* hold for the neutrino oscillation parameters, presented in Table 1. Therefore the mixing angle  $\vartheta_e^2$  cannot become zero, but has a non-trivial lower bound. To find the minimal value that it can reach, we consider the ratio of the angles  $\vartheta_e^2/(\vartheta_e^2 + \vartheta_\mu^2 + \vartheta_\tau^2)$ . Due to the unitarity of  $V$ , the denominator is

$$\sum_\alpha \vartheta_\alpha^2 \approx \frac{1}{2M_s^2} \sum_{\alpha,\beta,\gamma} V_{\alpha\beta}^* \tilde{M}_{D,\beta 2} V_{\alpha\gamma} \tilde{M}_{D,\gamma 2}^* = \frac{1}{2M_s^2} \sum_\beta |\tilde{M}_{D,\beta 2}|^2 = \frac{|z|^2}{4M_s} (m_2 + m_3). \quad (15)$$

Let us denote the ratio of the mixing of sterile neutrinos with one flavour to the sum of all mixings by  $T_\alpha$ ,

$$T_\alpha \equiv \frac{\vartheta_\alpha^2}{\sum_\beta \vartheta_\beta^2}. \quad (16)$$

Then we get the following expression for  $T_e$ :

$$T_e = \frac{|ie^{i\xi} c_{13} s_{12} \sqrt{\frac{m_2}{m_3}} - s_{13}|^2}{1 + \frac{m_2}{m_3}}. \quad (17)$$

The minimum is achieved if we push  $\theta_{12}$  and  $\Delta m_{21}^2$  to their  $3\sigma$  lower boundaries,  $\theta_{13}$  and  $\Delta m_{31}^2$  to their upper boundaries, and choose  $\xi = -\pi/2$ . The maximum is achieved when we set  $\Delta m_{31}^2$  equal to its lower bound,  $\theta_{13}, \theta_{12}$  and  $\Delta m_{21}^2$  to their upper bounds, and by choosing the Majorana phase  $\alpha = \pi/2$ . The bounds on  $T_e$  from Table 2 translate into the bounds for the muon and tau flavours combined:

$$0.83 \leq T_\mu + T_\tau \leq 0.9997. \quad (18)$$

The minimum and maximum of different  $T_\alpha$  are listed in the Table 2.<sup>5</sup> This analysis was conducted in approximation of large  $|z|$ . See Appendix C for the account of finite- $|z|$  effects.

### 3.3 Inverted hierarchy

Similarly to the previous case, for the inverted hierarchy we get a solution of the see-saw equations (7)

$$\tilde{M}_{D,i2} = iz \sqrt{\frac{M_s}{2}} (e^{-i\zeta} \sqrt{m_1}, ie^{-i\xi} \sqrt{m_2}, 0), \quad \tilde{M}_{D,i3} = iz^{-1} \sqrt{\frac{M_s}{2}} (e^{-i\zeta} \sqrt{m_1}, -ie^{-i\xi} \sqrt{m_2}, 0) \quad (19)$$

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<sup>5</sup>Notice that if one uses the results of [27] instead of [2, 3] to determine the upper and lower bounds on the neutrino mass matrix, the results, listed in the Table 2 can get modified (mostly, because the Ref. [27] provides broader interval for the possible values of  $\theta_{13}$ ). For example, it becomes possible to suppress completely  $\vartheta_e$  [17].

Normal hierarchy	Inverted hierarchy	Normal hierarchy	Inverted hierarchy
$1.7 \times 10^{-3} \leq T_e \leq 0.15$	$0.02 \leq T_e \leq 0.98$	$3 \times 10^{-4} \leq T_e \leq 0.17$	$0.02 \leq T_e \leq 0.98$
$0.09 \leq T_\mu \leq 0.89$	$3 \times 10^{-5} \leq T_\mu \leq 0.60$	$0.07 \leq T_\mu \leq 0.92$	$0 \leq T_\mu \leq 0.63$
$0.08 \leq T_\tau \leq 0.88$	$1.5 \times 10^{-5} \leq T_\tau \leq 0.62$	$0.06 \leq T_\tau \leq 0.90$	$0 \leq T_\tau \leq 0.65$
<i>The ranges are based on <math>2\sigma</math> bounds</i>		<i>The ranges are based on <math>3\sigma</math> bounds</i>	

Table 2: The ratio of the sterile neutrino mixing with a given flavour  $\alpha$  to the *sum* of the three mixings,  $T_\alpha$  (defined by (16)). **Left** table shows the upper and lower values of  $T_\alpha$  when parameters of neutrino oscillations are allowed to vary within their  $2\sigma$  boundaries (taken from [3]). The **right** table shows the results when the parameters of active neutrino oscillations are varied within their  $3\sigma$  limits (see Table 1). See the footnote 5 regarding the use of different  $3\sigma$  bounds.

for  $0 \leq \xi < 2\pi$ . In this case  $\vartheta_\mu^2$  or  $\vartheta_\tau^2$  can become very suppressed, as we will show soon.

The mixing angles are

$$\vartheta_\alpha^2 = \frac{|z|^2}{4M_s} \left| \sqrt{m_1} V_{\alpha 1} - i e^{i(\xi-\zeta)} \sqrt{m_2} V_{\alpha 2} \right|^2 + \frac{1}{4M_s |z|^2} \left| \sqrt{m_1} V_{\alpha 1} + i e^{i(\xi-\zeta)} \sqrt{m_2} V_{\alpha 2} \right|^2. \quad (20)$$

For  $|z| \gg 1$  they can become close to zero only if  $\sqrt{m_1} |V_{\alpha 1}| = \sqrt{m_2} |V_{\alpha 2}|$ . For  $\alpha = \mu$  this condition translates into

$$|\tan \theta_{12} + \sin \theta_{13} \tan \theta_{23} e^{-i\phi}| = \sqrt{\frac{m_2}{m_1}} |1 - \sin \theta_{13} \tan \theta_{12} \tan \theta_{23} e^{-i\phi}|. \quad (21)$$

For the parameter set close to the best fit, left-hand side is *less* than the right-hand side, because then  $\sin \theta_{13} \approx 0$ , while  $\tan \theta_{12} < 1$  and  $m_1 \approx m_2$ . To attain the equality one has to push left-hand side up and the right-hand side down.  $\phi = 0$  makes phases of both complex terms inside  $|\dots|$  on the left-hand side equal, thereby the absolute value of their sum becomes maximal. Simultaneously the right-hand side becomes minimal. For this specific choice of the Dirac angle the equality (21) turns into

$$\frac{\sqrt{\frac{m_2}{m_1}} - \tan \theta_{12}}{\sqrt{\frac{m_2}{m_1}} \tan \theta_{12} + 1} = \sin \theta_{13} \tan \theta_{23}. \quad (22)$$

The  $3\sigma$  bounds for inverted hierarchy in general are the same as for the normal one (see Table 1) with the exception of the “atmospheric” mass difference and  $\theta_{13}$  bound, that slightly differ. Using these values we find

$$0.14 < \frac{\sqrt{\frac{m_2}{m_1}} - \tan \theta_{12}}{\sqrt{\frac{m_2}{m_1}} \tan \theta_{12} + 1} < 0.24, \quad 0.03 < \sin \theta_{13} \tan \theta_{23} < 0.26. \quad (23)$$

We see that two regions overlap, therefore the relation (21) can be satisfied and  $\vartheta_\mu^2$  can be zero in a wide region of values of the parameters of the neutrino oscillation matrix. See, however, Sec. 3.4 below.

Similarly, the condition  $\vartheta_\tau = 0$  (for  $\phi = \pi$ ) translates into

$$\frac{\sqrt{\frac{m_2}{m_1}} - \tan \theta_{12}}{\sqrt{\frac{m_2}{m_1}} \tan \theta_{12} + 1} = \sin \theta_{13} \cot \theta_{23}, \quad (24)$$

and can be satisfied, because the quantity on the right hand side varies from 0.02 to 0.25, well within the range of (23).<sup>6</sup>

On the other hand,  $\vartheta_e$  can be zero only if

$$\cot \theta_{12} = \sqrt{\frac{m_2}{m_1}} \quad (25)$$

can be realized. The left hand side is always larger than the right hand side (within the  $3\sigma$  region), therefore no  $\vartheta_e$  suppression can occur. However it is important to know what minimal value this mixing angle can reach. According to Eq.(20) electron mixing angle is given by

$$\vartheta_e^2 = \frac{|z|^2}{4M_s} \cos^2 \theta_{13} (m_1 \cos^2 \theta_{12} + m_2 \sin^2 \theta_{12} + \sin(\xi - \zeta) \sin 2\theta_{12} \sqrt{m_1 m_2}). \quad (26)$$

For  $\xi - \zeta = -\pi/2$  this quantity is minimal

$$\vartheta_{e,min}^2 = \frac{|z|^2}{4M_s^2} \cos^2 \theta_{13} (\sqrt{m_1} \cos \theta_{12} - \sqrt{m_2} \sin \theta_{12})^2. \quad (27)$$

To compare it with the other mixing angles, we note that the relation

$$\sum_{\alpha} \vartheta_{\alpha}^2 \approx \frac{|z|^2}{4M_s} (m_1 + m_2) \quad (28)$$

holds (similar to Eq.(15) in the case of normal hierarchy). Therefore

$$\frac{\vartheta_{e,min}^2}{\sum_{\alpha} \vartheta_{\alpha}^2} = \frac{\cos^2 \theta_{13}}{1 + \frac{m_2}{m_1}} \left( \cos \theta_{12} - \sqrt{\frac{m_2}{m_1}} \sin \theta_{12} \right)^2. \quad (29)$$

The results of the analysis are listed in Table 2. From the upper bound on  $T_{\alpha}$  we derive the bound

$$T_{\mu} + T_{\tau} \geq 0.02. \quad (30)$$

We see that in this mass hierarchy it is possible for the overall coupling of the sterile neutrino to both  $\mu$  and  $\tau$  flavours to become tiny compared to the electron flavour coupling.

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<sup>6</sup>It was pointed out in [17] that both  $\vartheta_{\mu}$  and  $\vartheta_{\tau}$  can be suppress in inverted hierarchy, for  $\theta_{13} = 0$ . For this to happen the relation  $\sqrt{\frac{m_2}{m_1}} = \tan \theta_{12}$  should hold, as one can also see from Eqs. (22) and (24). The corresponding value of  $\theta_{12}$  is however well outside the  $3\sigma$  interval. The general case  $\theta_{13} \neq 0$  has not been analyzed in [17].



Flavour $\alpha$	$(\vartheta_\alpha^2)_{\min}$ @ 1 MeV	$ z $
$e$	$10^{-9}$	1.7
$\mu, \tau$	$10^{-8}$	1.5

(a) NH, best-fit

Flavour $\alpha$	$(\vartheta_\alpha^2)_{\min}$ @ 1 MeV	$ z $
$e$	$2 \times 10^{-10}$	5
$\mu, \tau$	$6 \times 10^{-9}$	1.8

(b) NH,  $3\sigma$ 

Flavour $\alpha$	$(\vartheta_\alpha^2)_{\min}$ @ 1 MeV	$ z $
$e$	$10^{-8}$	2.3
$\mu, \tau$	$2 \times 10^{-9}$	3.5

(c) IH, best-fit

Flavour $\alpha$	$(\vartheta_\alpha^2)_{\min}$ @ 1 MeV	$ z $
$e$	$6 \times 10^{-9}$	2.7
$\mu, \tau$	see text	

(d) IH,  $3\sigma$ 

Table 3: Minimal values of the active-sterile mixing angles  $\vartheta_\alpha^2$ , obtained using the best fit values of neutrino oscillation parameters (**left columns**) or by varying the neutrino oscillation data within their  $3\sigma$  intervals, listed in Table 1 (**right columns**). The values for  $(\vartheta_\alpha^2)_{\min}$  are provided for sterile neutrinos with the mass  $M_s = 1$  MeV. For other masses one should multiply the listed values by  $(\text{MeV}/M_s)$ . Columns “ $|z|$ ” show the values of  $|z|$  for which the minimum in (31) is reached.

### 3.4 Minimal mixing angles in the $\nu$ MSM

Finally, we find the *minimal* values of the sterile neutrino mixing angles in the  $\nu$ MSM, compatible with the neutrino oscillation data. These angles will turn out to be much smaller than the experimental upper bounds in all regions of masses, probed by the experiments. A general solution of the see-saw equations (12), (20) gives  $\vartheta$  as a function of  $|z|$ :

$$\vartheta_\alpha^2 = A_\alpha |z|^2 + \frac{B_\alpha}{|z|^2} \quad (31)$$

with coefficients  $A_\alpha$  and  $B_\alpha$  independent of  $|z|$ . The minimum of this expression is reached for  $|z|_\alpha^2 = \sqrt{B_\alpha/A_\alpha} \leq 1$  and is given by

$$(\vartheta_\alpha^2)_{\min} = 2\sqrt{A_\alpha B_\alpha}. \quad (32)$$

To find an *absolute* lower bound on the mixing angle for a given sterile neutrino mass, we vary this expression over the parameters of neutrino oscillations. The resulting mixing angles and the corresponding values of  $|z|$  are listed in Table 3.<sup>7</sup>

For the mixing angles  $\vartheta_{\mu,\tau}^2$  in the case of inverted hierarchy, different consideration takes place. Formally for infinitely large  $|z|$  they would become zero. The value of  $|z|$ , however, is bounded from above from the experiments  $|z| < z_{ub}$ . To see this, one should look at Fig. 7, where the resulting bounds are given. Therefore couplings to  $\mu$  and  $\tau$  neutrinos are finite for any phenomenologically plausible parameter set of the  $\nu$ MSM. The appropriate estimates of mixing angles can be provided for  $B_\alpha$  given by  $L_\alpha^{IH}$  (46,47), along with  $A_{\mu,\tau} = 0$ ,  $z = z_{ub}$

$$\vartheta_{\mu,\tau}^2 \gtrsim 2 \times 10^{-8} \frac{\text{MeV}}{M_s |z|_{ub}^2}, \quad (\text{IH}) \quad (33)$$

<sup>7</sup>Notice, that the ratios of the mixing angles  $\vartheta_\alpha^2/\vartheta_\beta^2$  do not reach their minima/maxima when the angles are minimized. The “interesting” values of  $|z|$  (those that can relax the bounds on the lifetime) are such that some of the mixing angles reach their upper experimental bounds.

## 4 Experimental bounds on sterile neutrino mixings

The direct experimental searches for neutral leptons had been performed by a number of collaborations [28–40] (see e.g. [10, 13] for review of various constraints). The negative results of the searches are converted into the *upper* bound on  $\vartheta_\alpha\vartheta_\beta$  for different flavours. If neutrino oscillations are mediated by these sterile neutrinos, these bounds can be translated into the *upper* bounds on parameter  $|z|$  and *lower* bounds on sterile neutrino lifetime.

Below, we take a closer look at two main types of experiments (“peak searches” and “beam-dump experiments”)<sup>8</sup> and describe *reinterpretation of these bounds* in the case, when sterile neutrinos with MeV–GeV masses are also responsible for neutrino oscillations.

### 4.1 Peak searches

In “*peak search*” experiments [43], one considers the two-body decay of charged  $\pi$  or  $K$  mesons to charged lepton ( $e^\pm$  or  $\mu^\pm$ ) and neutrino (see e.g. [10] for discussion). In case of the pion decay the limit on  $\vartheta_e^2$  for masses in the range  $60 \text{ MeV} \leq M_s \leq 130 \text{ MeV}$  is provided by the searches for the secondary positron peak in the decay  $\pi^+ \rightarrow e^+ N$  to the massive sterile neutrino  $N$  as compared to the primary peak coming from the  $\pi^+ \rightarrow e^+ \nu_e$  decay. Recent analysis of [39] puts this limit at  $\vartheta_e^2 < 10^{-8}$  in the mass range  $60 - 129 \text{ MeV}$ , for earlier results see [30, 31]. In the smaller mass region ( $4 \text{ MeV} \lesssim M_s \lesssim 60 \text{ MeV}$ ) Refs. [30, 31] provided the bound based on the change of the number of events in the primary positron peak located at energies  $M_\pi/2$ . Similar bounds were obtained for the same mixing angle in studies of *kaon* decays [34] and for the  $\vartheta_\mu^2$  in the decays of both pions [36–38] and kaons [34, 35].

The *lower* bound on the sterile neutrino lifetime  $\tau_s$  in the model (4), based on the peak search data and neutrino oscillations is shown in Fig. 1 by dot-dashed green lines. The parameters of neutrino mixing matrix are allowed to vary within their  $3\sigma$  limits (to minimize  $\tau_s$ , while still keeping the values of all mixing angles compatible with the bounds from direct experimental searches).

### 4.2 Beam-dump experiments and neutral currents contribution

The second kind of experiments (“*beam-dump experiments*”) [29, 32, 33] aims to create sterile neutrinos in decays of mesons and then searches for their decays into pairs of charged particles. Notice, that the expected signal in this second case is proportional to  $\vartheta_\alpha^4$  or  $\vartheta_\alpha^2\vartheta_\beta^2$  (and not to  $\vartheta_\alpha^2$  as in the case of peak searches, discussed in the Section 4.1). We will demonstrate below that that in the models like (4) (and in particular in the  $\nu\text{MSM}$ ) the results of some beam-dump experiments should be reinterpreted and will provide stronger bounds than discussed in previous works [10, 13, 17] (see also [44]).

### 4.3 Reinterpretation of the PS191 and CHARM experiments

The experiment **PS191** at CERN was a “beam-dump” type of experiment described above [28, 29]. In searches for sterile neutrinos lighter than the pion  $M_s < M_\pi$ , the pair of charged par-

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<sup>8</sup>Notice, that the neutrinoless double-beta decay ( $0\nu\beta\beta$ ) does not provide significant restrictions on the parameters of the sterile neutrinos in the type-I see-saw models (contrary to the case discussed in e.g. [10]), see discussion in [17, 41]. In particular, this is the case in the  $\nu\text{MSM}$  [42].

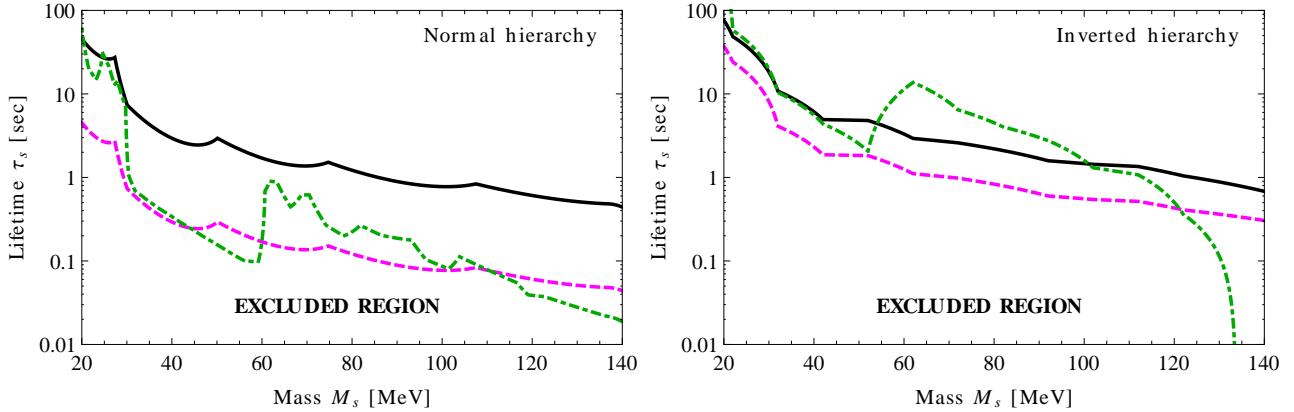


Figure 1: The lower bounds on the lifetime of sterile neutrinos, responsible for the mixings between active neutrinos of different flavours in the see-saw models (4). The bounds are based on the combination of negative results of direct experimental searches [28, 29, 31, 34–39] with the neutrino oscillation data [3]. The neutrino oscillation parameters are allowed to vary within their  $3\sigma$  confidence intervals to minimize the lifetime. The solid black curve is based on our reinterpretation of PS191 data *only*, that takes into account charged and neutral current contributions (see Sec. 4.3). The interpretation of the PS191 experiment, taking into account only CC interactions (used e.g. in the previous works [13, 17]) is shown in magenta dashed line. The bound from peak searches experiments *only* [31, 34–39] is plotted in green dot-dashed line.

ticles that were searched for in the neutrino decay comprised mostly of electron and positron:

$$\begin{aligned} \pi^+/K^+ &\rightarrow e^+ + N \\ &\hookrightarrow e^+ e^- \nu_\alpha, \end{aligned} \quad (34)$$

where  $N$  is a sterile neutrino with the mass  $M_s$ . The first reaction in the chain is solely due to the *charged-current* (CC) interaction, and its rate is proportional to the  $\vartheta_e^2$ .

If sterile neutrinos interact through both *charged and neutral currents* (CC+NC) as it is the case in the models with the see-saw Lagrangian (4), any of three active-neutrino flavours may appear in the decay of  $N$  in (34). The decay widths are [43]:

$$\Gamma(N \rightarrow e^+ e^- \nu_\alpha) = c_\alpha \vartheta_\alpha^2 \frac{G_F^2 M_s^5}{96\pi^3}, \quad (35)$$

with the following definition

$$c_e = \frac{1 + 4 \sin^2 \theta_W + 8 \sin^4 \theta_W}{4}, \quad c_\mu = c_\tau = \frac{1 - 4 \sin^2 \theta_W + 8 \sin^4 \theta_W}{4}, \quad (36)$$

and  $\theta_W$  is the Weinberg's angle so that  $\sin^2 \theta_W \approx 0.231$  and  $c_e \approx 0.59$ ,  $c_{\mu(\tau)} \approx 0.13$ . Therefore, the total number of events inside the detector that registers electron-positron pairs would be proportional to the combination of mixing angles  $\vartheta_e^2 \times (\sum c_\alpha \vartheta_\alpha^2)$ .

However, the model employed in the interpretation of the PS191 experiment [28, 29] was different, as has already been pointed in [44]. In the original analysis it was assumed that sterile neutrino interact *only via charged currents*, but not through neutral currents. In our

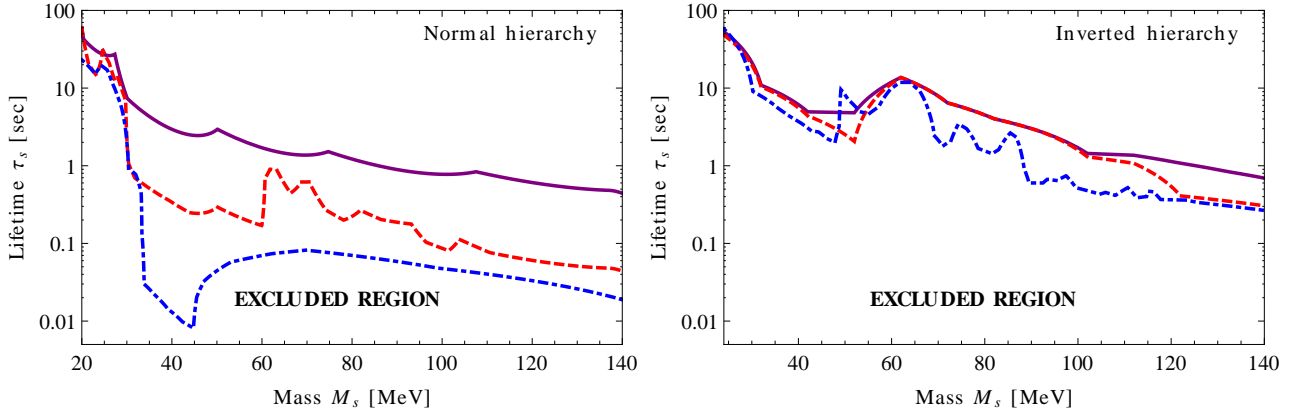


Figure 2: Comparison with the previous bounds on sterile neutrino lifetime in the  $\nu$ MSM [17]. The solid purple curves represent the results of the present work, obtained by the combination of peak searches experiments [31, 34–39] together with the reanalysis of PS191, that takes into account neutral currents (a union of black and green bounds from Fig. 1). The red dashed curve is based on the combination of the same peak searches with the *original* interpretation of PS191 (i.e., with charged currents only). The blue dot-dashed line is taken from [17]. Notice, that the results of [17] were multiplied by a factor 2 to account for the Majorana nature of the particles (see discussion in Sec. 4.4), that was missing therein. The difference between the red and blue lines in the case of normal hierarchy is explained by wider  $3\sigma$  intervals for neutrino oscillation data, used in [17], compared to our work.

language it means that  $c_e = 1, c_{\mu(\tau)} = 0$  was used instead of the values (36)<sup>9</sup>. As was noticed above, the probability of meson decay into sterile neutrino does not alter if we exclude the neutral-current interaction, and therefore the total number of events with the electron-positron pair would be proportional to  $\vartheta_e^2 \times \vartheta_e^2$ .

Therefore if we denote the bounds listed in [28, 29] as  $\vartheta_e^4 \leq \vartheta_{e(exp)}^4$ , then the bound for the  $\nu$ MSM takes form

$$\vartheta_e^2 \left( \sum_{\alpha=\{e,\mu,\tau\}} c_\alpha \vartheta_\alpha^2 \right) \leq \vartheta_{e(exp)}^4 . \quad (37)$$

Similar bounds can be extracted from the reanalysis of meson decays into *muon* and sterile neutrino, that leads to replacement  $e \rightarrow \mu$  in (37). As a result, the reinterpretation of the results of the PS191 experiment in combination with neutrino oscillation data produces up to an order of magnitude *stronger* bounds on lifetime than in the previous works (see Figs. 1 and 2).

Similarly, the CHARM experiment [33] provided bounds on the mixing angles of sterile neutrinos in the mass range  $0.5 \text{ GeV} \lesssim M_s \lesssim 2 \text{ GeV}$ . In the original analysis NC contributions *were neglected*. Therefore, to apply the results of this experiment to the case of the  $\nu$ MSM, we reanalyzed the data as described above. In Fig. 3 we compare lifetime bounds coming from the CHARM experiment solely for CC and CC+NC interactions of sterile neutrinos. The difference in this case is about a factor of 2.<sup>10</sup>

<sup>9</sup>Model described in [28, 29] contains only one Dirac neutrino, while in the  $\nu$ MSM we have two Majorana fermions. Therefore actually  $c_e = 1/2$  in the original model. For details see Sec. 4.4

<sup>10</sup>In the case of the PS191 experiment, when using CC only for masses below the mass of pion suppression of the  $\vartheta_e^2$  mixing angle due to neutrino oscillations meant that instead of  $\vartheta_e^2$  bounds the lifetime is defined by

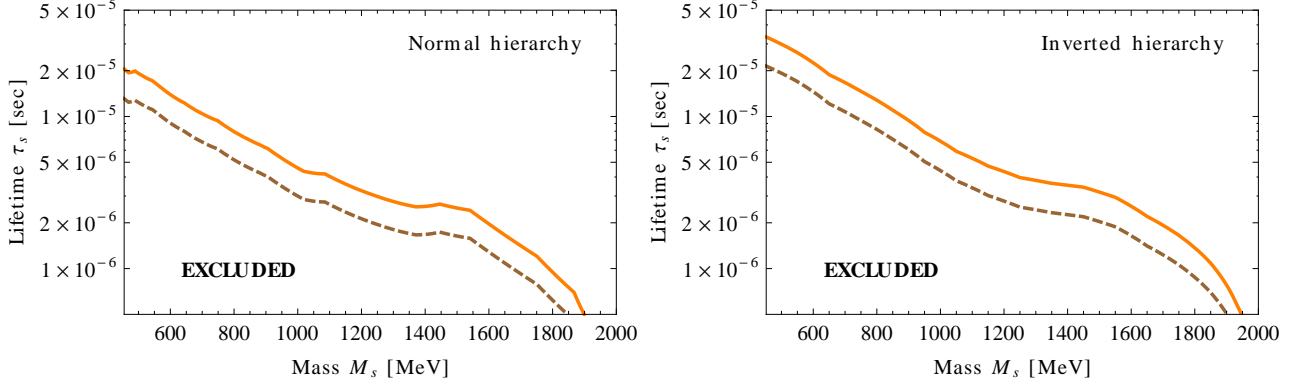


Figure 3: Comparison of the bounds on sterile neutrino lifetime (in the model (4)) based on the results of the CHARM experiments [33] *solely* (combined with the neutrino oscillation data). The orange (upper) curves correspond to the model with charged and neutral current interactions of sterile neutrinos, the brown (lower) – to the model with charged current interactions only. For details, see Sec. 4.3.

#### 4.4 A note on Majorana vs Dirac neutrinos

For completeness we briefly discuss the difference in interpreting experimental results for *Majorana vs. Dirac sterile neutrinos*. Similar discussion can be found e.g. in [13]. When interpreting the experimental results one should take into account that in present work we consider *two Majorana sterile neutrinos*, while the experimental papers often phrase their bounds in terms of the mixing with a single *Dirac* neutrino, that we will denote  $U_\alpha^2$ . In the  $\nu$ MSM twice more sterile neutrinos are produced per single reaction (because there are two sterile species –  $N_2$  and  $N_3$ ), and, owing to their Majorana nature, each sterile neutrino decays twice faster (additional charge-conjugated decay modes are present). Notice, that the mass splitting between two sterile states  $N_2, N_3$  is small  $|M_2 - M_3| \ll \frac{1}{2}(M_2 + M_3) = M_s$  and once born, the states oscillate fast into each other. Averaging over many oscillations can be accounted for by an extra factor  $\frac{1}{2}$  in the number of  $N_2$  and  $N_3$  species. Therefore, for beam-dump experiments one gets the same number of the detector events involving one Dirac sterile neutrino as one gets in the  $\nu$ MSM if  $(\vartheta_{\alpha 2}^2 + \vartheta_{\alpha 3}^2)^2 = U_\alpha^4$ . That is, one should identify  $2\vartheta_\alpha^2$  with the measured  $U_\alpha^2$  (recall (11) that  $\vartheta_\alpha^2 = \frac{1}{2}(\vartheta_{\alpha 2}^2 + \vartheta_{\alpha 3}^2)$ ). In the case of peak searches, the bound  $U_\alpha^2$  should be interpreted in the  $\nu$ MSM as  $\vartheta_{\alpha 2}^2 + \vartheta_{\alpha 3}^2 \leq U_\alpha^2$ , as production of *any* state  $N_2$  or  $N_3$  contributes to the number of events in the secondary peak, i.e. again  $2\vartheta_\alpha^2$  should be identified with  $U_\alpha^2$ . Notice, that this factor 2 is missing in [17].

## 5 Results

In this Section we summarize our results: the upper bound on the (combination of) mixing angles of sterile and active neutrinos in the see-saw models (2) in the range 10 MeV – 2 GeV

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the (much weaker)  $\vartheta_\mu^2$  bounds. That led to the significant relaxation of the lower bound on the lifetime. If NC were taken into account, this was not possible anymore and therefore the lower bound on sterile neutrino lifetime became stronger by as much as the order on magnitude (black vs. magenta curve on the left panel in Fig. 1. In case of the CHARM experiment, both  $\vartheta_e^2$  and  $\vartheta_\mu^2$  are strongly constrained and switching from one constraint to another makes (numerically) much smaller difference.

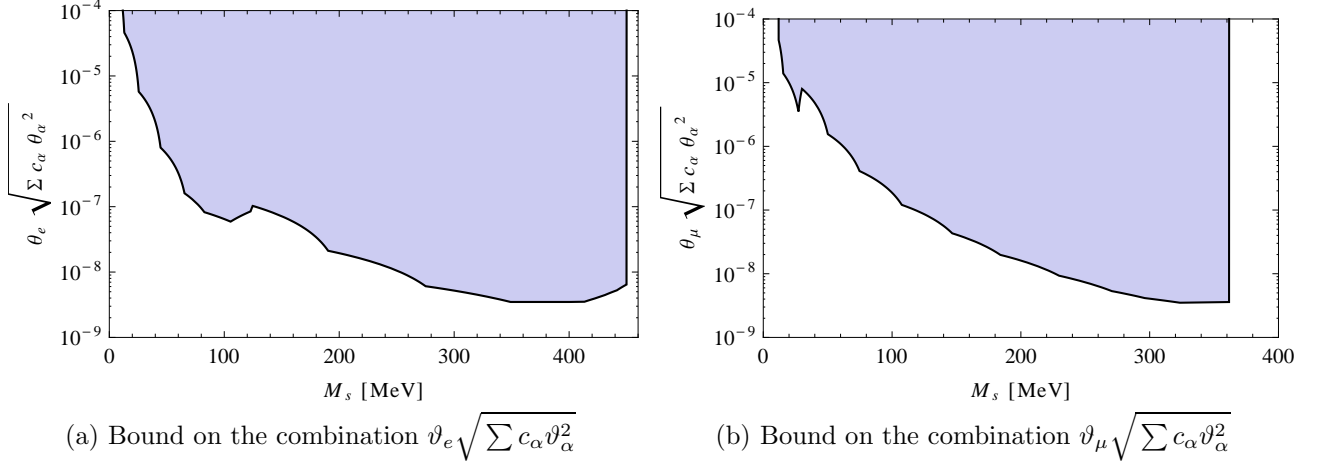


Figure 4: Direct accelerator bounds on the combination of active-sterile neutrino mixing angles, resulting from the reanalysis of the PS191 experiment [28, 29], taking into account decays of sterile neutrino through both charged and neutral currents and their Majorana nature. The shaded region is excluded. The case, analyzed in the original works [28, 29] (decay of sterile neutrino through the charged current only) corresponds to the choice  $c_e = 1$ ,  $c_\mu = c_\tau = 0$ , for details, see Sec. 4.3. We plot the bounds for *two Majorana* neutrinos (as in Fig. 5) while in the original works [28, 29] a single Dirac neutrino was analyzed.

and the lower bound on sterile neutrino lifetime, obtained in combination of these bounds with constraints, coming from neutrino oscillation data.

## 5.1 Bounds on the mixing angles of sterile neutrinos

For the models (4) (two Majorana sterile neutrinos, interacting through both charged and neutral interactions), the compilation of constraints on various combinations of active-sterile mixing angles ( $\vartheta_e^2$ ,  $\vartheta_\mu^2$ ,  $\vartheta_e \sqrt{\sum c_\alpha \vartheta_\alpha^2}$ ,  $\vartheta_\mu \sqrt{\sum c_\alpha \vartheta_\alpha^2}$ ) that we used in this work are plotted in Figs. 4 and 5.<sup>11</sup>

## 5.2 The lower bound on the lifetime of sterile neutrinos

The result of the Sections 3.2–3.3, combined with these experimental bounds can be translated into the *lower* limits on the lifetime of sterile neutrinos. These results are presented in Figs. 6 on page 16. One sees that for the masses below that of pion the lifetime bound can be relaxed by as much as an order of magnitude (see the upper right panel in Fig. 6).

# 6 Conclusion

In this work we have investigated experimental restrictions on the parameters of the see-saw Lagrangian in the case when two sterile neutrinos with the masses between  $\sim 10$  MeV and

<sup>11</sup>Notice that in the published results of the PS191 experiment [29] bounds are given up to  $M_s = 400$  MeV. We extend these bounds up to 450 MeV, using the PhD Thesis of J.-M. Levy [45].

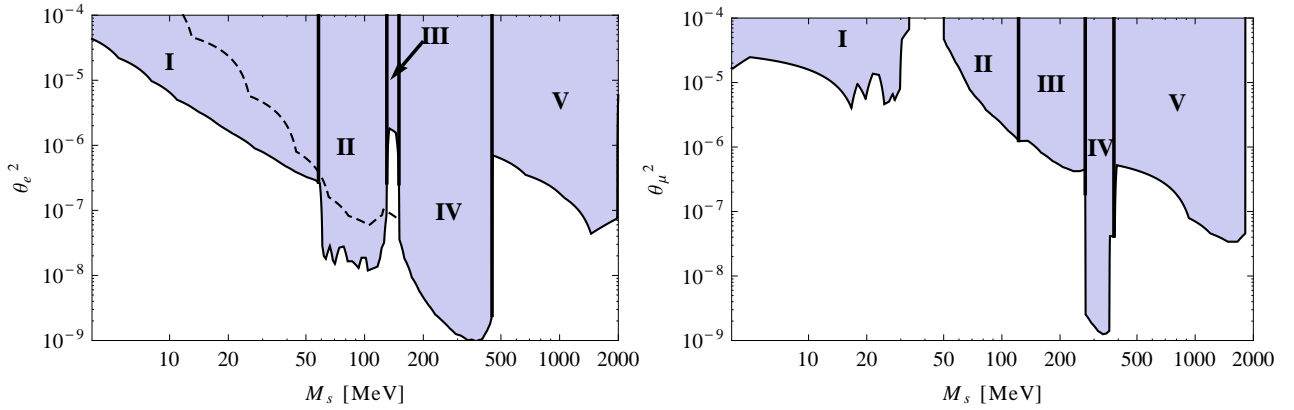


Figure 5: Direct accelerator bounds on the mixing angles. **Left panel:**  $\vartheta_e^2$  bounds, taken from [31] (region I), [39] (region II), [34] (region III), [29, 45] (region IV) and [33] (region V). **Right panel:**  $\vartheta_\mu^2$  bounds, taken from [36–38] (region I), [34] (region II), [35] (region III), [29] (region IV) and [32] (region V). The shaded regions are ruled out by the experimental findings. Dashed curve indicates mixing angle bounds given by original interpretation of PS191 experiment, but we *do not* use it to derive our final results, as explained in Sec. 4.3. The bounds are shown for the Majorana neutrino and are therefore two times *stronger* (see Section 4.4), while in the original works [28, 29] a single Dirac neutrino has been considered.

2 GeV, responsible for neutrino oscillations. Combined with the results of the direct experimental searches, the neutrino oscillation data provide stringent lower bounds on their lifetime,  $\tau_s$  and allows to determine both *maximum* and *minimum* values of the mixing angles  $\vartheta_\alpha^2$ .

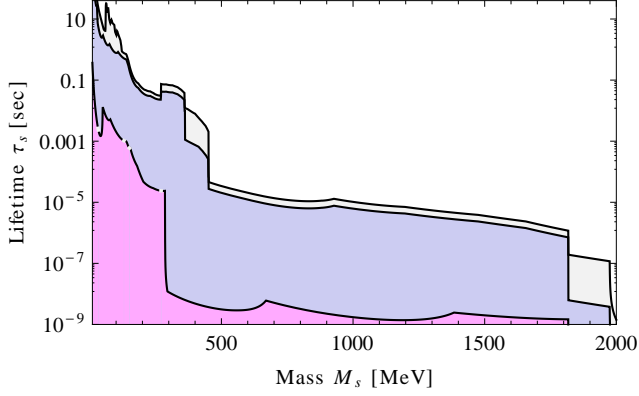
It was argued in [17] that peak searches in the pion decay “*might miss sterile neutrinos since the branching ratios can be extremely small due to the cancellation in  $\vartheta_e^2$ , but the search of  $K^+ \rightarrow \mu^+ + N$  can cover the whole allowed region*”. As we have demonstrated in this work, the full suppression of  $\vartheta_e^2$  does not occur if the most recent data on neutrino oscillation experiments is taken into account. Moreover, contrary to the [17] we conclude that the complete suppression of the mixing angle  $\vartheta_\mu^2$  may occur in the case of inverted hierarchy (Section 3.3). Our results imply that the peak searches in kaon decay may miss the sterile neutrino as well.

We have reinterpreted the results of the PS191 experiment [28, 29], following [44] by taking into account not only charged-, but also neutral-current interactions (as both of these are present in the Type I see-saw Lagrangian). Our results demonstrated that in the low mass region (below the mass of the pion) the beam-dump experiments ( $\vartheta^4$  experiments) provide *stronger* restrictions than the peak search experiments ( $\vartheta^2$  experiments) in case of *normal hierarchy*. In *inverted hierarchy* the reanalysis of the PS191 experiment turns out to be very important as well. Also we point out that in the original analysis of the CHARM experiment [32] neutral-current contributions were neglected as well. We have reinterpreted the results of this experiment in a similar way to PS191. The final results are presented in Figs. 6.

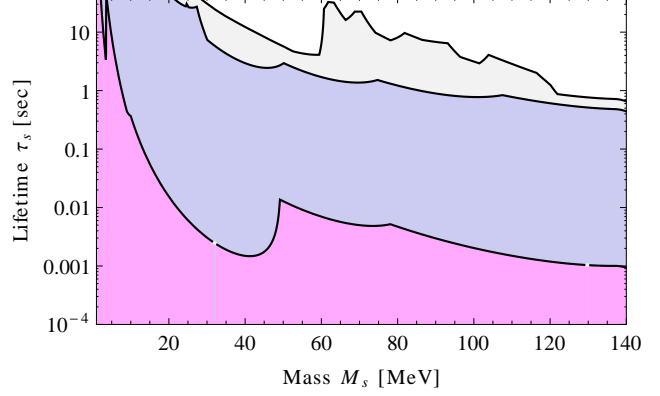
Future experiments (for example, NA62 in CERN<sup>12</sup>, LBNE experiment in FNAL<sup>13</sup> or the LHCb experiment) have a great potential of discovering light neutral leptons of the  $\nu$ MSM or significantly improving the bounds on their parameters (see e.g. [46]).

<sup>12</sup><http://na62.web.cern.ch/NA62>

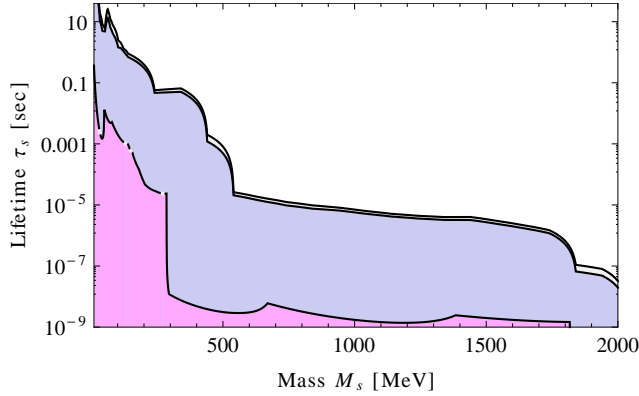
<sup>13</sup><http://lbne.fnal.gov>



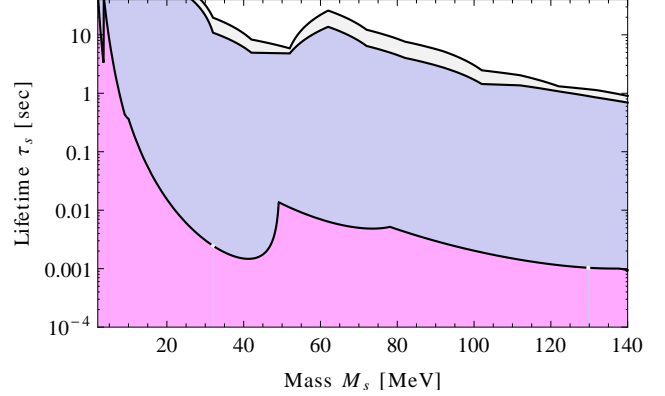
(a) Normal hierarchy, mass range 10 MeV – 2 GeV



(b) Normal hierarchy, zoom at the mass range 10MeV – 140 MeV



(c) Inverted hierarchy, mass range 10 MeV – 2 GeV



(d) Inverted hierarchy, zoom at the mass range 10MeV – 140 MeV

Figure 6: The resulting lower bounds on sterile neutrino lifetime  $\tau_s$  as a function of their mass, obtained by requiring that two Majorana sterile neutrinos are responsible for neutrino oscillations and their parameters do not contradict the negative results of direct experimental searches. In all figures the upper curve comes from using of the best fit neutrino oscillation parameters, the middle one – from their variation within the  $3\sigma$  limits, and the lower one does not take into account neutrino oscillation data and puts all three mixing angles equal to their direct experimental values.



Finally, we notice that sterile neutrinos with the masses in MeV–GeV region and small mixing angles can play significant role in the early Universe (for review see [14]). Because of their small mixing angles these particles can be out-of-thermal equilibrium in the early Universe. In particular they can lead to the successful baryogenesis scenario [21, 24, 47]; generate large lepton asymmetry at temperatures below the sphaleron freeze-out [23]. In Fig. 7 we superimpose the bounds on  $|z|$ , coming from the direct experimental searches on the region of parameters  $(|z|, M_s)$  in which the successful baryogenesis is possible (the region inside the black contours marked “BAU” based on the Ref. [24]). Additionally, these sterile neutrinos can generate large lepton asymmetry at  $T \lesssim \text{few GeV}$  [23] that can affect production of dark matter sterile neutrinos [48, 49]. Their lifetime can be long enough to be present in primordial plasma at  $\sim \text{MeV}$  temperatures, so they can influence the primordial synthesis of light elements [12] and produce additional entropy and energy during their decays at later times [50]. Sterile neutrinos with the mass of about 200 MeV and mixing angles  $\sim 10^{-7} - 10^{-8}$  can affect the physics of supernova explosions [51]. Finally, our constraints may affect the low-reheating temperature cosmological scenario (see e.g. [52, 53]).

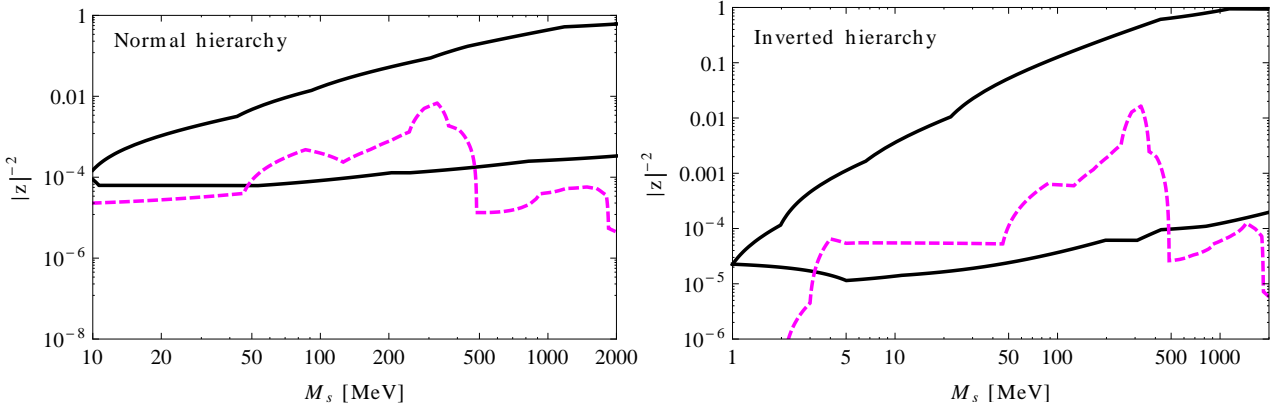


Figure 7: The region of successful baryogenesis in the  $\nu\text{MSM}$  compared with the experimental lower bounds on the parameter  $|z|$ . The values of  $M_s$  and  $|z|$ , lying inside the black solid lines lead to the production of the observable baryon asymmetry (from [24]). The magenta dashed line marks the *lower* bound on  $|z|^{-2}$  (parameter, called  $\epsilon$  in [23, 24]) such that for smaller values at least one of the mixing angles  $\vartheta_\alpha^2$  is in contradiction with direct experimental searches (under any assumptions about the values of unknown parameters of the PMNS matrix).

## Acknowledgments.

We would like to thank T. Asaka, F. Bezrukov, A. Boyarsky, S. Eijima, D. Gorbunov, H. Ishida, S. Pascoli, D. Semikoz, M. Shaposhnikov for discussion and useful comments. A.I. is also grateful to S. Vilchynskiy, Scientific and Educational Centre of the Bogolyubov Institute for Theoretical Physics in Kiev, Ukraine<sup>14</sup> and to Ukrainian Virtual Roentgen and Gamma-Ray Observatory VIRGO.UA.<sup>15</sup> The work of A.I. was supported in part from the Swiss-Ukrainian cooperation project (SCOPES) No. IZ73Z0-128040 of Swiss National Science Foundation.

<sup>14</sup><http://sec.bitp.kiev.ua>

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## A Sterile neutrino lifetime

For any phenomenologically interesting mass of the sterile neutrino it can decay to three neutrinos  $N \rightarrow \nu\bar{\nu}\nu$  and it results in contribution

$$B_{\nu\nu\nu}^e = B_{\nu\nu\nu}^\mu = B_{\nu\nu\nu}^\tau = 1 \quad (38)$$

to the total sum in Eq. (2) over decay products  $X$ .

If  $M_s > 2M_e \approx 1.0$  MeV, then a new decay channel appears:  $N \rightarrow e^+e^-\nu$ . It corresponds to

$$\begin{aligned} B_{ee\nu}^\alpha = & \left( \frac{1}{4} \pm \sin^2 \theta_W + 2 \sin^4 \theta_W \right) \left[ (1 - 14S_e^2 - 2S_e^4 - 12S_e^6) \sqrt{1 - 4S_e^2} \right. \\ & + 12S_e^4 (S_e^4 - 1) \ln \left( \frac{1 - 3S_e^2 - (1 - S_e^2)\sqrt{1 - 4S_e^2}}{S_e^2(1 + \sqrt{1 - 4S_e^2})} \right) \Big] \\ & + 2 \sin^2 \theta_W (2 \sin^2 \theta_W \pm 1) \left[ S_e^2 (2 + 10S_e^2 - 12S_e^4) \sqrt{1 - 4S_e^2} \right. \\ & + 6S_e^4 (1 - 2S_e^2 + 2S_e^4) \ln \left( \frac{1 - 3S_e^2 - (1 - S_e^2)\sqrt{1 - 4S_e^2}}{S_e^2(1 + \sqrt{1 - 4S_e^2})} \right) \Big] , \end{aligned}$$

where  $S_X = M_X/M_s$ ,  $\theta_W$  is the Weinberg angle, and sign "+" corresponds to  $\alpha = e$  while "-" in the other case. For  $S_\mu < \frac{1}{2}$  we should add up to a sum terms  $B_{\mu\mu\nu}^\alpha$  that can be obtained from  $B_{ee\nu}^\alpha$  if one changes  $S_e \rightarrow S_\mu$  and if sign "+" corresponds to  $\alpha = \mu$ , "-" - to  $\alpha = e, \tau$ .

Other  $B$ -coefficients are

$$B_{e\mu\nu}^{e,\mu} = 1 - 8S_\mu^2 + 8S_\mu^6 - S_\mu^8 - 24S_\mu^4 \ln S_\mu, \quad B_{e\mu}^\tau = 0, \quad (S_\mu + S_e < 1), \quad (39)$$

$$B_{\pi\nu}^\alpha = 6\pi^2 \frac{f_\pi^2}{M_s^2} (1 - S_\pi^2)^2, \quad (S_\pi < 1, \quad M_\pi \approx 140 \text{ MeV}, \quad f_\pi \approx 130 \text{ MeV}), \quad (40)$$

$$B_{\eta\nu}^\alpha = 6\pi^2 \frac{f_\eta^2}{M_s^2} (1 - S_\eta^2)^2, \quad (S_\eta < 1, \quad M_\eta \approx 550 \text{ MeV}, \quad f_\eta \approx 155 \text{ MeV}), \quad (41)$$

$$B_{\pi e}^e = 12\pi^2 \frac{f_\pi^2}{M_s^2} V_{ud}^2 \left( (1 - S_e^2)^2 - S_\pi^2 (1 + S_e^2) \right) \sqrt{(1 - (S_\pi - S_e)^2)(1 - (S_\pi + S_e)^2)}, \quad (S_e + S_\pi < 1), \quad (42)$$

$$B_{\pi\mu}^\mu = 12\pi^2 \frac{f_\pi^2}{M_s^2} V_{ud}^2 \left( (1-S_\mu^2)^2 - S_\pi^2(1+S_\mu^2) \right) \sqrt{(1-(S_\pi-S_\mu)^2)(1-(S_\pi+S_\mu)^2)}, \quad (S_\mu+S_\pi < 1). \quad (43)$$

In the last expression quantity  $V_{ud} \approx 0.97$  is the element of the Cabibbo-Kobayashi-Maskawa (CKM) quark matrix. Expression for  $B_{Ke}^e$ , that is non-zero for mass range  $S_e + S_K < 1$  ( $M_K \approx 495$  MeV), can be derived from  $B_{\pi e}^e$  by simultaneous change  $S_\pi \rightarrow S_K, V_{ud} \rightarrow V_{us} \approx 0.23, f_\pi \rightarrow f_K \approx 160$  MeV. Moreover, if we change in the resulting expression  $S_e$  by  $S_\mu$ , we get  $B_{K\mu}^\mu$  (for  $S_\mu + S_K < 1$ ). What concerns B-coefficients with  $\alpha = \tau$  and  $X = (\pi e, \pi\mu, Ke, K\mu)$ , they are all zero. For masses of sterile neutrino, that are larger than the mass of the  $\rho$ -meson  $M_\rho \approx 780$  MeV, other  $B_X$  appear. We do not list them in the present paper, however they can be read from [13].

## B PMNS parametrization

In Section 3 we use the standard definition of the PMNS matrix (cf. e.g. [1]):

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & e^{i\phi} & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12}e^{i\phi} - c_{12}s_{13}s_{23} & c_{12}c_{23}e^{i\phi} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{23}s_{12}e^{i\phi} - c_{12}c_{23}s_{13} & -c_{12}s_{23}e^{i\phi} - c_{23}s_{12}s_{13} & c_{13}c_{23} \end{pmatrix}. \quad (44)$$

where  $c_{ij} = \cos \theta_{ij}$ , and  $s_{ij} = \sin \theta_{ij}$ . The active neutrino mixing matrix is given by expression (5).

## C Ratio of sterile neutrino mixing angles for $|z| \sim 1$

As the expressions (12) and (20) show, the mixing angles have two terms: one is proportional to  $|z|^2$  and another to  $|z|^{-2}$  (recall that  $|z| \geq 1$ ). It was shown in Sec. 3.3 that for inverted hierarchy the  $|z|^2$ -term can be zero for  $\vartheta_\mu^2$  and  $\vartheta_\tau^2$ , while the  $|z|^{-2}$  term in general stays finite.

For a given value of  $|z|$ , the  $|z|^{-2}$ -term is bounded from above. According to (12) and (20), its maximum is realized simultaneously with the maximal value of

$$L_\alpha^{NH} = |V_{\alpha 3} + ie^{i\xi} \sqrt{\frac{m_2}{m_3}} V_{\alpha 2}|^2 \quad (45)$$

in the normal hierarchy, and

$$L_\alpha^{IH} = |V_{\alpha 1} + ie^{i(\xi-\zeta)} \sqrt{\frac{m_2}{m_1}} V_{\alpha 2}|^2 \quad (46)$$

in the inverted hierarchy.

Analysis, similar to that of the Sections 3.2–3.3 shows that

$$L_e^{NH} \leq 0.2, \quad L_\mu^{NH} \leq 1.1, \quad L_\tau^{NH} \leq 1.1, \quad L_e^{IH} \leq 1.96, \quad L_\mu^{IH} \leq 1.3, \quad L_\tau^{IH} \leq 1.3. \quad (47)$$

These bounds allow to estimate the contribution of the  $|z|^{-2}$ -terms to the whole sum of the squared mixing angles

$$\sum_{\alpha} \vartheta_{\alpha}^2 = \frac{m_1 + m_2 + m_3}{4M_s} \left( |z| + \frac{1}{|z|} \right) . \quad (48)$$

The ratio of (45)–(46) to (48) gives

$$R_{\alpha}^{NH} = \frac{L_{\alpha}^{NH}}{(1 + \frac{m_2}{m_3})} \frac{1}{|z|^4 + 1}, \quad R_{\alpha}^{IH} = \frac{L_{\alpha}^{IH}}{(1 + \frac{m_2}{m_1})} \frac{1}{|z|^4 + 1} . \quad (49)$$

For  $z \sim 1$  it can become of order unity. However, we restrict ourselves to the sufficiently large values  $z \gtrsim 10$ , that are consistent with the upper bound, indicated by the experiments (see Fig.7)

$$R_e^{NH} \lesssim 2 \times 10^{-5}, \quad R_{\mu,\tau}^{NH} \lesssim 10^{-4}, \quad R_e^{IH} \lesssim 2 \times 10^{-4}, \quad R_{\mu,\tau}^{IH} \lesssim 10^{-4} . \quad (50)$$

Comparison these results with the *lower* bounds (Table 2) we see that  $z^{-1}$  terms are unimportant for the for all mixing angles in NH and  $\vartheta_e$  in IH. What concerns the remaining angles  $\vartheta_{\mu}$  and  $\vartheta_{\tau}$  in IH, they can be substantially modified by account of  $z^{-1}$ -terms, but anyway each of them can become small enough, compared to the other angles, as explained in next section. As a corollary, analysis and results of Secs. 3.2,3.3 do not change significantly for large enough values of  $z$ .